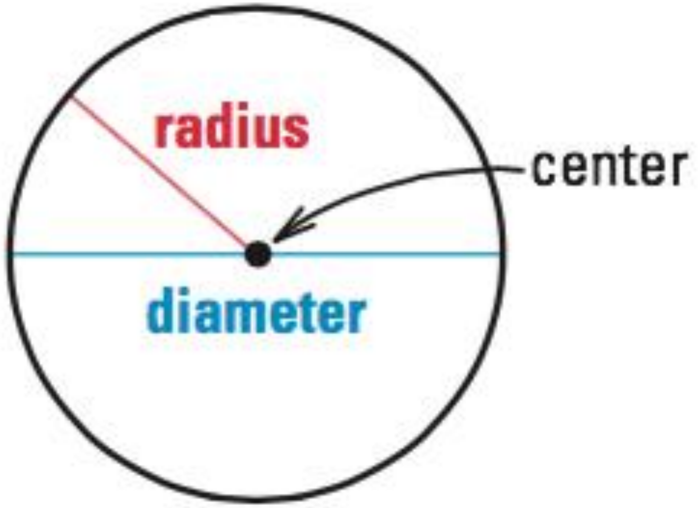
Chapter 10
Circles

Section 1
Tangents to Circles

GOAL 1: Communicating About Circles

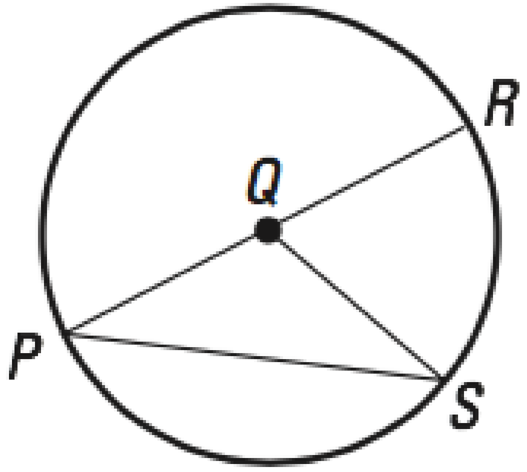
A **circle** is the set of all points in a plane that are equidistant from a given point, called the **center** of the circle. A circle with center *P* is called "circle *P*", or $\odot P$.



The distance from the center to a point on the circle is the **radius** of the circle. Two circles are **congruent** if they have the same radius.

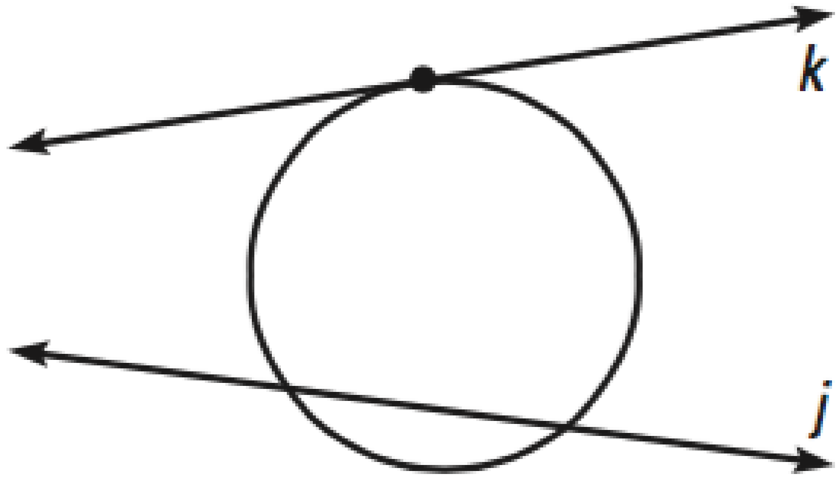
The distance across the circle, through its center, is the **diameter** of the circle. The diameter is twice the radius.

The terms *radius* and *diameter* describe segments as well as measures. A **radius** is a segment whose endpoints are the center of the circle and a point on the circle. $\overline{QP}$, $\overline{QR}$, and $\overline{QS}$ are radii of $\odot Q$ below. All radii of a circle are congruent.



A **chord** is a segment whose endpoints are points on the circle. $\overline{PS}$ and $\overline{PR}$ are chords.

A **diameter** is a chord that passes through the center of the circle. $\overline{PR}$ is a diameter.

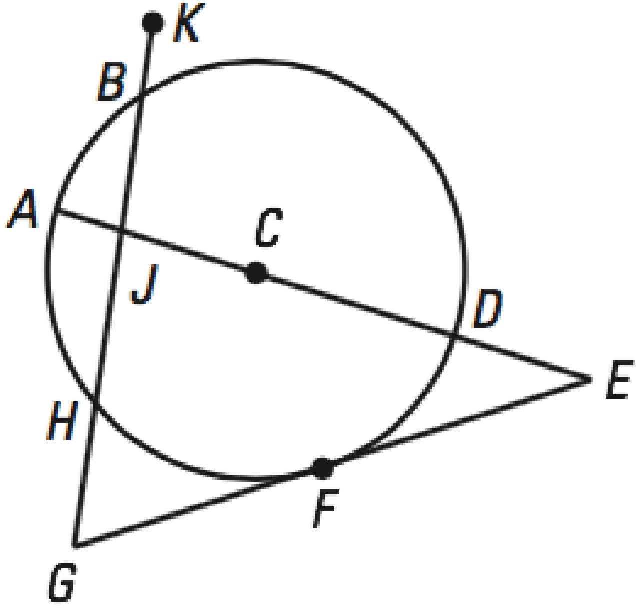


A **secant** is a line that intersects a circle in two points. Line j is a secant.

A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point. Line k is a tangent.

Example 1: Identifying Special Segments and Lines

Tell whether the line or segment is best described as a chord, a secant, a tangent, a diameter, or a radius of Circle C.



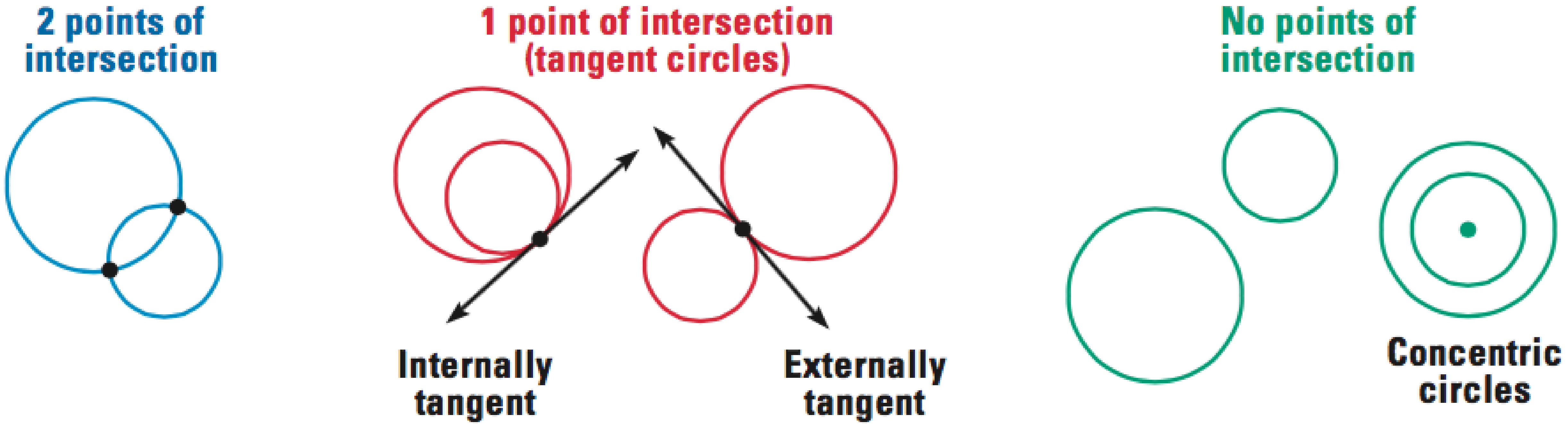
a) AD → diameter

b) CD → radius

c) EG → tangent

d) HB → chord (GK → secant)

In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric**.

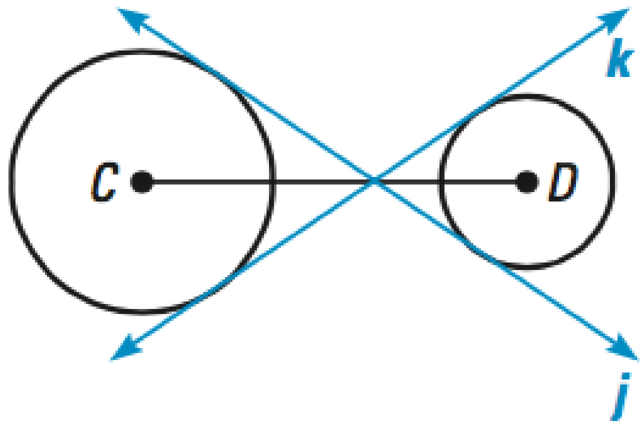


A line or segment that is tangent to two coplanar circles is called a **common tangent**. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

Example 2: Identifying Common Tangents

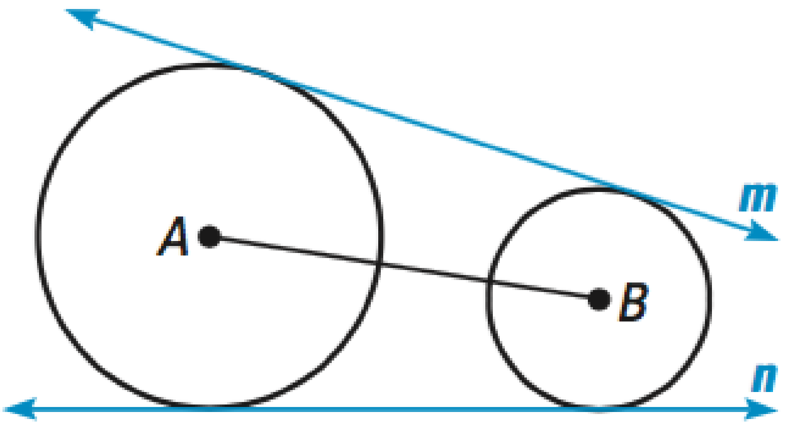
Tell whether the common tangents are internal or external.

a.



internal

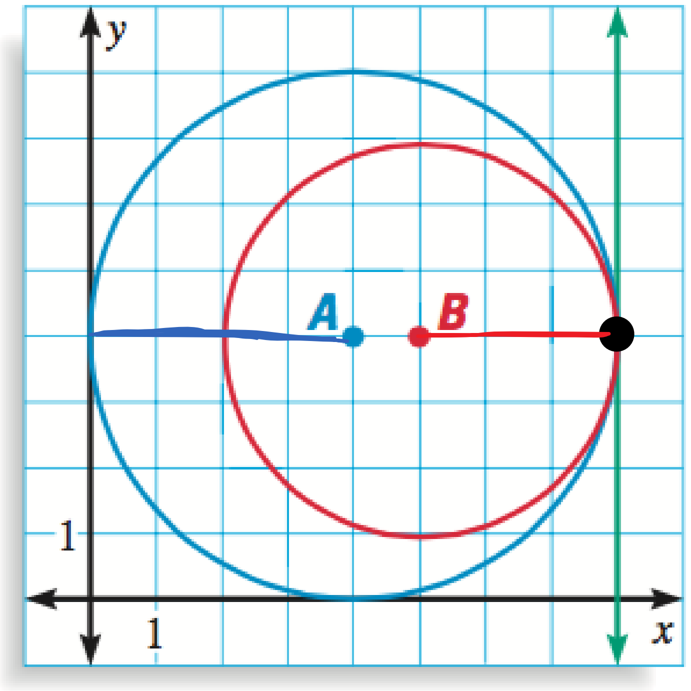
b.



external

In a plane, the **interior of a circle** consists of the points that are inside the circle. The **exterior of a circle** consists of the points that are outside the circle.

Example 3: Circles in Coordinate Geometry



Give the center and the radius of each circle. Describe the intersection of the two circles and describe all common tangents.

A: (4, 4) r = 4

B: (5, 4) r = 3

Intersect @ (8, 4)

Common tangents: x = 8 (green line)

GOAL 2: Using Properties of Tangents

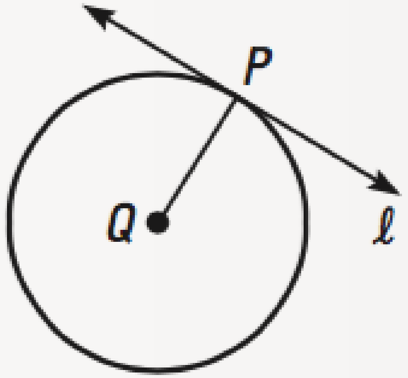
The point at which a tangent line intersects the circle to which it is tangent is the **point of tangency.** You will justify the following theorems in the exercises.

THEOREMS

THEOREM 10.1

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

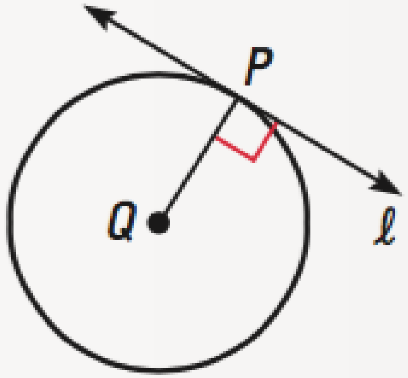
If ℓ is tangent to $\odot Q$ at P, then $\ell \perp \overline{QP}$.



THEOREM 10.2

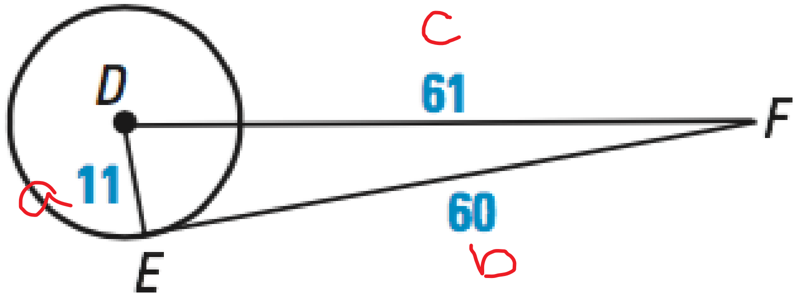
In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

If $\ell \perp \overline{QP}$ at P, then ℓ is tangent to $\odot Q$.



Example 4: Verifying a Tangent to a Circle

You can use the Converse of the Pythagorean Theorem to tell whether EF is tangent to Circle D.



$c^2 \overset{?}{=} a^2 + b^2$

$61^2 \overset{?}{=} 11^2 + 60^2$

$3721 = 3721$

right $\angle$

$\Rightarrow \perp \Rightarrow$ tangent

Example 5: Finding the Radius of a Circle

You are standing at C, 8 feet from a grain silo. The distance from you to a point of tangency on the tank is 16 feet. What is the radius of the silo?

$$(r+8)^2 = r^2 + 16^2$$

$$\cancel{r^2} + 16r + 64 = \cancel{r^2} + 256$$

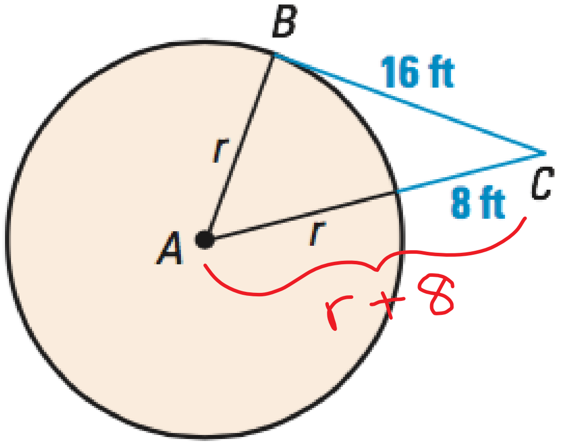
$$-r^2 \qquad -r^2$$

$$16r + \cancel{64} = 256$$

$$-\cancel{64} \qquad -64$$

$$\frac{\cancel{16}r}{\cancel{16}} = \frac{192}{16}$$

$$r = 12$$



$$(r+8)^2$$

$$(r+8)(r+8)$$

F
O
I
L

$$r^2 + 8r + 8r + 64$$

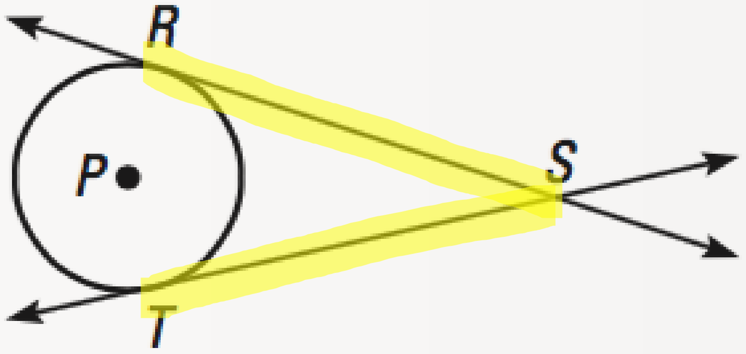
$$r^2 + 16r + 64$$

From a point in a circle's exterior, you can draw exactly two different tangents to the circle. The following theorem tells you that the segments joining the external point to the two points of tangency are congruent.

THEOREM

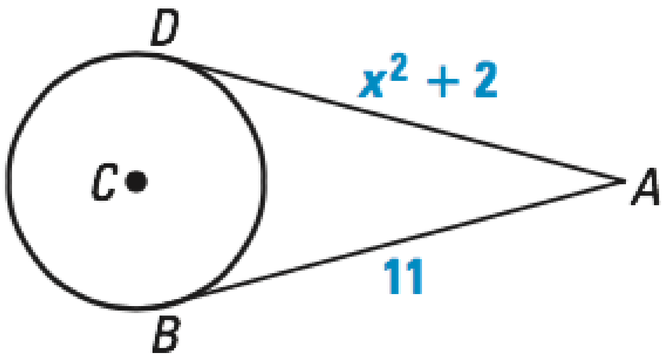
THEOREM 10.3

If two segments from the same exterior point are tangent to a circle, then they are congruent.



If $\overleftrightarrow{SR}$ and $\overleftrightarrow{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

Example 7: Using Properties of Tangents



AB is tangent to Circle C at B. AD is tangent to Circle C at D.

Find the value of x.

$$x^2 + \cancel{2} = 11$$
$$\quad\; -\cancel{2} \quad -2$$

$$\sqrt{\cancel{x^2}} = \sqrt{9}$$

$$x = 3$$

EXIT SLIP